
Exploring Odds and Evens

Key Question: Why are there so few odd products?

Task

- 1) Each member of the group estimates how many of the 59 products are even.
- 2) The group sorts the cards to establish there are 20 odd and 39 even products.
- 3) The group investigates the two piles to work out why there are so few odd products.

Solution

To get an odd product you need to multiply two odd factors. For example:

$$5 \times 3 = 15$$

$$7 \times 9 = 63$$

Explanation

The other possible combinations of odd and even products all produce an even product.

Even factor \times Even factor = Even product

Odd factor \times Even factor = Even product

Even factor \times Odd factor = Even product

Since multiplication is commutative, odd \times even = even \times odd.

In other words there are really only three possible combinations of odd and even factors.

Odd \times odd represents 1 out of the 3 possibilities so you would expect the number of products which are odd to be about one third of the total number of products. One third of 59 is indeed approx. 20.

Times Tables Cards

Exploring Odds and Evens Cont.

This investigation requires children to reason mathematically. It should also help them self-correct when trying to retrieve a times table answer.

For example, after the investigation they should know that the answer to 7×9 could not be 62 or 64.

Differentiation

Students with excellent tables knowledge can be asked to do the investigation without access to the cards. This will require them to survey all the numbers from 1 - 144 to identify products from the times tables up to 12×12 .

They will then have to think mathematically rather than relying on observation of the cards to deduce the reason why there are so few odd products.

Exploring Doubles within a Table

Key Question: How many doubles can you find within a times table?

Task

- 1) The group chooses any times table they know.
- 2) They lay out the multiples of that times table, up to $12x$.
- 3) The group spots any multiples which are double another multiple.

Solution

Whichever times table the group picks, they will discover that there are 6 doubles.

For example in the $4x$ table the doubles look like this:

4 doubles to 8
8 doubles to 16
16 doubles to 32
20 doubles to 40
12 doubles to 24
24 doubles to 48

This investigation gives children doubling practice. But it also reinforces conceptual understanding of what tables are. While doing the investigation children revisit the concept that tables represent a number of equal groups, and that if you double the number of groups you double the product.

Times Tables Cards

Exploring Doubles within a Table Cont.

Extension

Obviously, there are also 6 halves within any times table. Ask the children to pick a different times table, to lay out the multiples in a line and then identify the 6 halves.

Differentiation

Students with excellent tables knowledge can be challenged to practise saying their tables in a new order following the families of doubles:

$$1 \times 3 = 3$$

$$3 \times 3 = 9$$

$$5 \times 3 = 15$$

The rest

$$2 \times 3 = 6$$

$$6 \times 3 = 18$$

$$10 \times 3 = 30$$

$$7 \times 3 = 21$$

$$4 \times 3 = 12$$

$$12 \times 3 = 36$$

$$9 \times 3 = 27$$

$$8 \times 3 = 24$$

$$11 \times 3 = 33$$

Exploring Multiples of 3

Key Question: How do you spot a multiple of 3?

Task

Pose the two questions:

- 1) How many cards show 3 as a factor?
- 2) How many cards have 3 as a factor?

You could also pose the questions as:

- 1) What fraction of the cards show 3 as a factor?
- 2) What fraction of the cards have 3 as a factor?

Solution

To answer the first question, children can just sort through the cards looking for any showing 3 as a factor. Alternatively they can deduce what they think the answer should be, and then check.

The answer is 11 cards show 3 as a factor (11/59)

Children might expect the answer to be 12, but 3 is the first of the 12 multiples.

The cards do not show any factor pairs where 1 is a factor, so 3 does not show 1 and 3 as factors.

Times Tables Cards

Exploring Multiples of 3 Cont.

To answer the second question, children can do a number of things:

- (i) short division
- (ii) write out the multiples of 3 beyond 36 and match to cards
- (iii) recognise that any multiple of 6, 9 or 12 will also be a multiple of 3
- (iv) work out the digital root for all products

Let children find their own method, then discuss alternatives and compare.

The answer is 29 cards have 3 as a factor (29/59)

Digital Roots

This is a very easy and useful technique for spotting multiples of 3.

A digital root is the single digit value you reach by (repeatedly) adding the digits of a number.

For example:

72: digital root is $7 + 2 = 9$

121: digital root is $1 + 2 + 1 = 4$

Where your initial addition > 9 , add the digits of the new number.

56: $5 + 6 = 11$

11: $1 + 1 = 2$

Digital root of 56 is 2

Any multiple of 3 will have a digital root which is a multiple of 3. In other words, the digital root of a multiple of 3 is 3 or 6 or 9.

Exploring Square and Cube Numbers

Key Questions:

- 1) What are the common features of square numbers?
 - 2) How many of the square numbers are also cube numbers?
 - 3) How many products are cube but not square?
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Task

- 1) Sort through all the cards and find any cards which show an odd number of factors. Identify what the cards all have in common.
- 2) Look at these cards and work out which of these cards show a product which is also a cube number.
- 3) Work out if there are any other products in the pack which are cube numbers.

Solution

- 1) When children sort through the pack they should find that the only products with an odd number of factors are the square numbers.

It is good for children to notice that where a factor pair is made up of two numbers the same, these factors are shown on the sides of the cards. This embeds the idea that a square number is what you get when you multiply a number by itself. (All other factor pairs are shown at the top and bottom of the cards.)

2) and 3)

Once children have grasped the concept of a square number, it is easy to introduce cube numbers as a number multiplied by itself and by itself again.

Let children find as many cube numbers as they can. Don't remind them that the highest product in the pack is 144! Let them do lots of cube number calculations and then search for those answers in the pack.

Once they have a list of cube numbers they can answer part 2 and 3.

1 and 64 are both square and cube ($1 \times 1 \times 1 = 1$ and $4 \times 4 \times 4 = 64$)

8 is a cube number ($2 \times 2 \times 2$). So is 27 ($3 \times 3 \times 3$).

Extension

How many products in the pack are both square and numbers to the power of four.
The answer is 1, 16, 81