

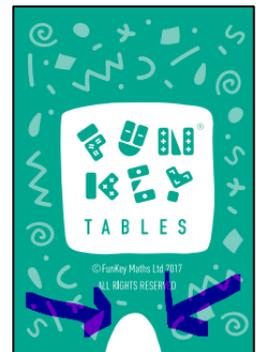
Concept 1: Activities to develop basic conceptual understanding of multiplication and division

These notes are the first part of the FunKey guide to making the most of your Times Tables cards. They describe a progressive sequence of activities which you can undertake either whole class or in an intervention setting to develop foundational conceptual understanding of the meaning of simple multiplication and division equations. Since our focus is conceptual understanding, the examples we use are from the full range of times tables and are not restricted to working with multiples of 2, 5 or 10. These activities provide a secure foundation upon which to build future learning of any times tables.

We strongly recommend that you complete these activities **before** working with pupils to build fluent recall of multiplication facts. When pupils have worked through these activities and have got basic conceptual understanding of the meaning of a multiplication fact, you can move onto the activities suggested in [Recall 1](#). Recall 1 contains a progressive sequence of activities to help pupils develop fluent recall of key multiplication facts.

Resources needed

- Counters – ideally double-sided
- Counter worksheet – Step 3
- FunKey Times Table Cards – 1 pack per group
- FunKey Times Table Card Holder – 1 per group
- Bar model and part whole model worksheets – Step 10
- Number line worksheet – Step 11
- Five standard dice
- Five Dice score sheet – Step 12



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Before you start

Ensure you have numbered your times tables cards! There is a small white space on the backs of the cards for numbering cards. Give all cards in the same pack the same number/letter. Use a permanent pen. If you don't do this, you will waste hours trying to work out which pack the card on the floor belongs to...



To make the job of managing the cards in class easier, use a subset of 16 cards for these activities. In the first part of this teaching sequence you will be working with the cards with these numbers in the middle:

3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 18, 20, 22, 24

Store the 16 cards in the FunKey card holder and leave the rest of the pack in the box. If the cards are numbered, it is easy enough to recombine packs when this teaching sequence is complete.

We use counters and representations of counters in this teaching sequence. As pupils become more confident in their understanding of the multiplicative relationship, other representations will be introduced.

Multiplication is introduced in primary schools as the composition of a whole from equal parts. Division is the decomposition of a whole into equal parts. This approach simplifies multiplication and division down to three key elements:

- *How many parts?*
- *How big is each part?*
- *How big is the whole?*

As the activities below start with counters, we are going to start with the following three key questions to describe groups of counters.

- *How many groups?*
- *How many counters in each group?*
- *How many counters altogether?*

To develop understanding it is critically important in the first instance that pupils respond in full sentences. So, for this image the key questions and answers would be:



Three key questions

Three key answers

How many groups?

There are three groups.

How many counters in each group?

There are two counters in each group.

How many counters altogether?

There are six counters altogether.

Once children are fluent in answering the three key questions, the next stage is to contract the question and answer by removing the object – counters, as follows:



Three key questions

Three key answers

How many groups?

There are three groups.

How many in each group?

There are two in each group.

How many altogether?

There are six altogether.

Once children are fluent in answering the three key questions with three key answers in the contracted format, the next stage is to help them describe the counters in just two sentences, in the following format.

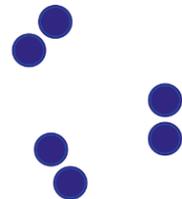
The summary question

Contracted two sentence answer

What can you see?

*There are **three groups of two**.*

There are six altogether.



By the end of Step 3 we want pupils to know what three groups of two means. In other words, if they are asked to make three groups of two, they can do it with counters, and if they see three groups of two counters, they can say it is three groups of two and explain the meaning of three and two in this context.

In these activities, we use “spatially distinct” groups. By this we simply mean something which looks like the image to the right, rather than an array.



An array of six counters shows both three groups of two and two groups of three whereas the image to the right is unambiguously two groups of three. The ambiguity of arrays is confusing to many pupils who are new to multiplication, so we use an unambiguous representation in the first instance to clarify meaning.



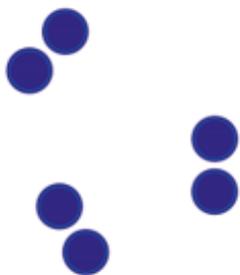
Later, as multiplicative thinking develops, arrays do have an important role, particularly in teaching commutativity and bridging to the hugely helpful area model. Arrays will be introduced in the activity sequence Concept 2 which we are publishing soon! Look out for it on [@FunKeyMaths](#)

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Step 1: asking and answering the three key questions with counters

If teaching whole class, show pupils a representation of spatially distinct equal groups of counters on the whiteboard and ask pupils to make what they can see on their desks. [There is a whiteboard resource prepared for Step 1 here.](#) If working with a small group of pupils, use counters on the desktop to show them a spatially distinct set of equal groups.

Model the three key questions and the three key answers:



<u>Three key questions</u>	<u>Three key answers</u>
<i>How many groups?</i>	<i>There are three groups.</i>
<i>How many counters in each group?</i>	<i>There are two counters in each group.</i>
<i>How many counters altogether?</i>	<i>There are six counters altogether.</i>

Keep modelling this language until the pupils are confident using it. They will need lots of practice. Repeat the activity with some or all of these:

- | | |
|----------------|----------------|
| 5 groups of 3 | 6 groups of 3 |
| 2 groups of 6 | 10 groups of 2 |
| 4 groups of 4 | 2 groups of 7 |
| 2 groups of 10 | 1 group of 8 |

When the children are confidently answering the three key questions accurately without referring to counters, introduce the contracted two sentence answers.

<u>The summary question</u>	<u>Contracted two sentence answer</u>
<i>What can you see?</i>	<i>There are three groups of two. There are six altogether.</i>



Keep modelling this language until the pupils are confident. Again, they will need lots of practice.

Next, reverse the activity. You give them information using the contracted form, and the pupils represent it with counters.

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Step 2: making commutative facts with counters

As in Step 1 show a spatially distinct set of equal groups on the whiteboard, or, if working in a small group, make a spatially distinct set of equal groups with counters.

Ask the three key questions and ensure that when responding, pupils use the three key answers practised in Step 1 and the contracted answers.

Now **show the pupils the commutative fact**. Either move the counters on the desk in front of your intervention group or [use this whiteboard resource](#). Ask the pupils to say what they can see. Establish that you have the same number of counters, but they have been grouped in a different way. Notice the link between 2 groups of 3 and 3 groups of 2.

Repeat the activity with some of the following until pupils are able to predict confidently the commutative fact and show it with counters.

3 groups of 8	9 groups of 2
4 groups of 7	2 groups of 5
6 groups of 3	4 groups of 5
5 groups of 5	7 groups of 3

At this stage, you can use the words *commutative fact* and/or *commutativity* if you want to. If you prefer you can simply call the commutative fact the related multiplication fact. The label is not important. The key thing is that pupils see and then experience commutativity by playing with counters and arranging and rearranging them into equal groups.

Repeat the activity but this time tell the pupils one of the facts above and ask them to make the commutative fact with their counters.

When you are doing this whole class, it is difficult to monitor the accuracy of the language being used in each partner group. We highly recommend using older peer mentors to sit with pairs of pupils as they do these activities. [Click here for more information about our peer mentoring resources](#).

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Step 3: moving from counters to representations of counters

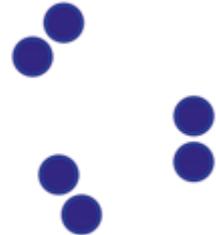
[Use the Step 3 worksheet.](#) Pupils will work in pairs. [Use the whiteboard resource](#) to model to pupils what to do with the worksheet. Ask pupils to look at each representation in turn and answer the three key questions. After modelling the activity whole class ask the pupils to work in pairs asking and answering the three key questions.

Repeat the activity but this time model using the summary question and responding with the contracted form.

<u>The summary question</u>	<u>Contracted two sentence answer</u>
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What can you see?

*There are **three groups of two**.
There are six altogether.*



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Checkpoint 1: things to know about the cards before using them in class

[A full explanation of the design features of the FunKey cards can be found here.](#)

The colours

Each times table has its own unique colour. Pupils soon learn the colours and those with good visual memories start to associate numbers with colours which can help them recall multiplication and division facts.

The antennae and matching background pattern

These are designed to help pupils who don't see colour to link multiples from the same times table.

The position of the numbers

The larger number in the middle is the whole, also known as the *product*. The smaller numbers around the edge of the card are *factors* of the *product*.

Notice that when two *factors* are the same, they appear at the side of the card, not at the top or bottom.

The shapes around the middle number

There are three different white shapes which appear around the *product*.

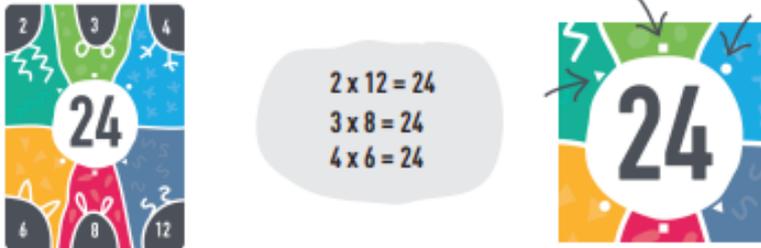
Squares: these are for *square numbers*

Kites: these are for *prime numbers*

Circles: these are for all other products

The tiny white shapes around the edge of the white shape in the middle

The tiny white shapes on cards with more than two factors help pupils match factor pairs. The general rule on four factor cards is to match diagonally, but on 24 and 36 it's more complicated and the little shapes can really help.



What's not on the cards!

There are no division or multiplication symbols and no equals sign so pupils use their understanding of the multiplicative relationships between numbers to create a range of different equations.

Not all factors of the middle number are shown on the cards. You will see all the factors you need to learn your times table facts up to 12×12 , excluding the $1 \times$ table. We don't show 1 as a factor on any card as 1 is a factor of every whole number, and we don't show the product in the middle of the card as a factor either as it is already on the card.



(That's not quite true! 1 is shown as a factor of 1, but that is to draw attention to the fact that 1 is a square number! Remember: 1 is not a prime number).

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Step 4: exploring the FunKey Times Table Cards

It's now time to introduce a new representation: FunKey Times Table Cards.

Working in pairs or threes, give each small group of pupils one of your pre-prepared card holders with the sub-set of cards outlined [above](#). Tell them to spread the cards out between them and discuss with their partners:

What do you notice?

What do you wonder?

Take feedback as a class, ensuring you draw attention to the following features of the cards:

- Colours and how they relate to each factor
- The shape in the middle of each card surrounding the product
- The position of the factors on the cards

Pupils may also comment on which factors are shown and which are not. It is worth mentioning to pupils at some point that not all the factors are shown, to avoid misconceptions.

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Step 5: using counters to explore two-factor cards

By now your pupils will be confident in making and describing equal groups of counters and will be familiar with the FunKey Times Tables Cards. It's time to match these two representations and introduce written maths code (multiplication and division equations).

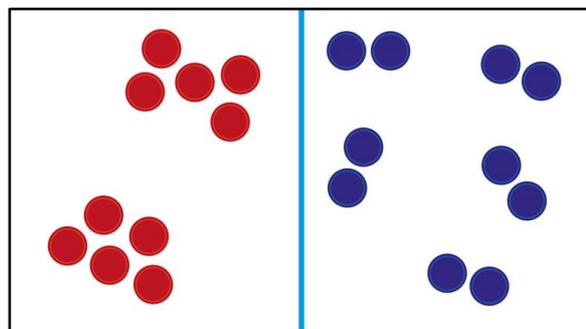
Ask pupils to take ten counters. Challenge them to split the ten counters up into equal groups. When they have completed that challenge, ask them to do it a different way using another 10 counters.

They will end up with something like this.

Ask the three key questions.

Say the contracted forms.

Revisit commutativity.



Of course, some pupils may also discover that 10 can be put into 1 group of 10 or into 10 groups of 1. This is good, but not essential for the next steps, so you do not need to encourage or discourage it. If it does come up, you can note that a group of counters can always be put into a single group or split up into single counters.

Repeat the activity with a different number of counters. Try exploring some of these numbers: 8, 9, 10, 14, 15, 21, 22.

When pupils have been through all the numbers listed and created different ways to split each of the numbers into equal groups, explore each number again, but this time get the matching Times Table Card out too.

Lay the card out next to the two different ways to arrange the counters (at this stage exclude 1 group of and groups of 1).



Talk about what is on the card and what each number on the card means.

Draw attention to the three key elements for the red counters and the blue counters. Notice what's the same and what's different.

You might say:

On the red side, there are 5 groups. There are 2 in each group. There are 10 altogether.
Point to the numbers on the card as you say the numbers.

Then say:

On the blue side, there are 2 groups. There are 5 in each group. There are 10 altogether.
Again, point to the numbers on the card as you say the numbers.

Start to introduce the maths code for writing about this.

$$5 \times 2 = 10$$

$$2 \times 5 = 10$$

We recommend that when you are verbalizing a multiplication equation, you read it like this. This is a further language contraction, which combines the two sentences in the first contracted form into a single sentence.

$$5 \times 2 = 10 \quad 5 \text{ groups of } 2 \text{ is } 10$$

$$2 \times 5 = 10 \quad 2 \text{ groups of } 5 \text{ is } 10$$

(We will explore a second way of talking about multiplication equations in Concept 2).

It is not important at this stage that pupils are able to write equations independently. You just want to make them aware of the symbols and enable them to link the symbols to their knowledge about equal groups.

Now repeat this activity using other cards from the card holder until pupils can confidently match equal groups of counters to the correct FunKey Times Table Card and they can explain the relationship between the counters and cards using the language modelled above. (Remember, they do not need to be able to write their own equations at this stage.)

So far, the language we have used starts with groups and ends with the total.



Next, do the exercise again, but when describing what they can see, ask pupils to start with the product and end with the groups.

You might say:

I have 10 counters.

On the red side, I have split 10 counters into 5 groups of 2 counters.

On the blue side, I have split 10 counters into 2 groups of 5 counters.

Start to introduce the maths code for writing about this, and practise how to talk about the division equation.

$10 \div 2 = 5$ (10 can be split up into groups of 2. There are 5 groups.)

$10 \div 5 = 2$ (10 can be split up into groups of 5. There are 2 groups.)

(We will explore a second way of talking about division equations in Concept 2.)

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Step 6: using counters to explore prime number cards

We can now introduce the pupils to the idea that some products have special properties. Repeat Step 4, but this time explore each of these products in turn:

3, 5, 7, 11

How many different ways are there to split the product into equal groups? The pupils will notice that the only possible ways to do it involve 1 group, or groups of 1.

Once you have made groups of counters showing 1 group and groups of 1, find the matching card. Pupils will notice the absence of factors around the edges of the cards. Notice too, the shape around the product has changed. Use the term prime number and discuss that with a prime number you can either leave the counters as one group, or put them into groups of 1. There is no other way to split the counters up into equal groups.

As with commutativity, it is not important at this stage that pupils can use the word “prime,” but it is good to introduce the label for the concept that they have observed.

You might say:

The only way to make 11 is 11 groups of 1 or 1 group of 11. So, 11 is a prime number.

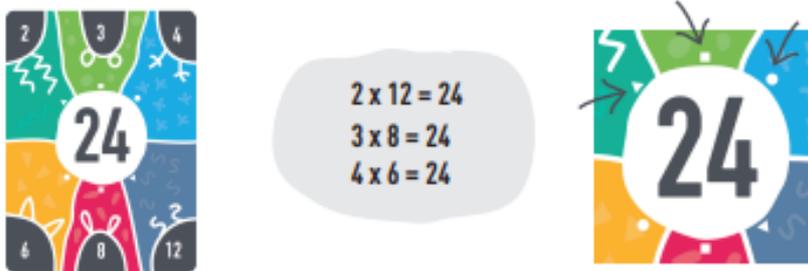
You might also want to rehearse writing maths code to match the sentences you create.

$11 \times 1 = 11$ *11 groups of 1 is 11*
 $1 \times 11 = 11$ *1 group of 11 is 11*
 $11 \div 1 = 11$ *11 can be split up into groups of 1. There are 11 groups.*
 $11 \div 11 = 1$ *11 can be split up into groups of 11. There is 1 group.*

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Step 7: using counters to explore cards with > 2 factors

Pupils are now ready to explore cards which have more than two factors. If you haven't already done so, point out to pupils the little white shapes on cards with more than two factors which help match factor pairs.



Again, repeat Step 4, but this time explore each of these products in turn, using counters:

12, 16, 18, 20, 24

How many different ways are there to split the product into equal groups? Ask pupils to find different ways to split counters up into equal groups. Ask pupils to leave all the groupings visible on their desks, and then find the card which matches what they have made.

Talk about all the different ways to make the product using the stem sentences and match these to written equations.

2×6	6×2
$12 \div 2$	$12 \div 6$



3×4	4×3
$12 \div 3$	$12 \div 4$

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Step 8: play Slap Bingo

This game works well with one person acting as the caller and up to 5 pupils playing against each other. However, when the game is new, play with a whole class with the teacher as caller and pupils sharing cards in pairs or small groups. Pupils lay all the cards from the card holder out in front of the players in random order, face up. The caller says a times table fact using the contracted form.

For example, the caller shouts:

5 groups of 2

7 groups of 3

4 groups of 4

The other players race to slap the right card. The player who slaps it first keeps it. The winner is the one with the most cards, when all the cards have been slapped.

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Checkpoint 2: change the card set in your cardholder

For Step 9 onwards you will need to add the following cards to the card holder:

27, 28, 35, 36, 44, 50, 63, 84

and you can remove the following cards:

3, 5, 7, 11, 16

The new set should be as follows:

6, 8, 9, 10, 12, 14, 15, 18, 20, 22, 27, 28, 35, 36, 44, 50, 63, 84

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Step 9: interpreting cards without counters

Now that pupils are confident making and describing groups of counters, matching them to the correct FunKey card and interpreting matching multiplication and division equations, the next step is for them to practise this knowledge without the support of having physical counters to move around.

Model to the pupils how to interpret a card **without** using counters to explore the product using the language of both multiplication and division as practised in previous activities.

Point to the relevant numbers on the card as you model. The pointing will support pupils in seeing the connections between factors and products.

Start with the groups

If I have 5 groups of 10 counters, I have 50 counters altogether.

If I have 10 groups of 5 counters, I have 50 counters altogether.

Start with the product

I have 50 counters

I can split them into 5 groups of 10 counters.

I can split them into 10 groups of 5 counters.

Don't miss the opportunity to write up the maths equations too!

[You may find this video helpful.](#) It shows two Year 3 pupils interpreting the meaning of Card 50. Firstly, they describe in sentences what the numbers represent, then they create the multiplication and division equations.

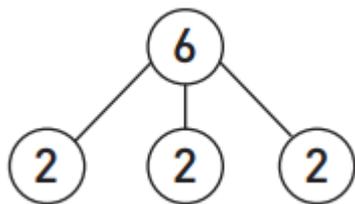
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Step 10: working with part-whole models and bar models

Your pupils should now be confident in recognizing and describing spatially distinct groups of counters as a representation for multiplication and division. Now it's time to introduce two new representations: part-whole models and bar models.

Step 10.1

Display a complete part-whole model on the whiteboard. [You can use the Step 10.1 power point.](#)



Discuss what the pupils notice about the representation. Model how to interpret it.

Use the words, *part* and *whole*.

Draw attention to the change in language: instead of talking about groups, we are now talking about parts and wholes. The three key questions and answers alter slightly:

How many parts? There are three parts.

How many in each part? There are two in each part.

How many in the whole? The whole is six.

A further contraction can now be introduced, as follows:

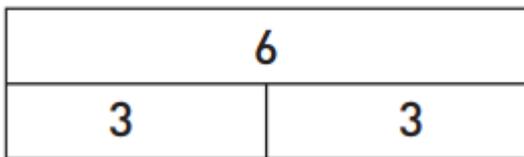
There are three twos. There are six altogether.

[Now find the worksheet Concept 1/ Step 10.1.](#) Discuss how to complete the models.

When this is done, ask the pupils to find the FunKey card which matches the model. Don't forget that for each pair of factors there will be two possible models. Pupils who are developing a more secure understanding of the relationship between products and factors can draw the other model.

Step 10.2

Once pupils are familiar with the part-whole diagram to represent the multiplicative relationship, they can be introduced to bar models. Display a complete part-whole model on the whiteboard. [You can use the Step 10.2 power point.](#)



Discuss what the pupils notice about the representation. Compare it to a part-whole model. What is similar, what is different? Model the language for talking about a bar model. Use the words, *part* and *whole*.

[Now find the worksheet Concept 1/ Step 10.2.](#) Discuss how to complete the models. You can also ask pupils to find the FunKey card which matches the model. Don't forget that for each pair of factors there will be two possible models. Pupils who are developing a more secure understanding of the relationship between products and factors can draw the other model.

As a final check of pupils' learning, show them a card, and ask them to draw the matching part-whole models or bar models. Remember that there will be two possible models for each factor pair.

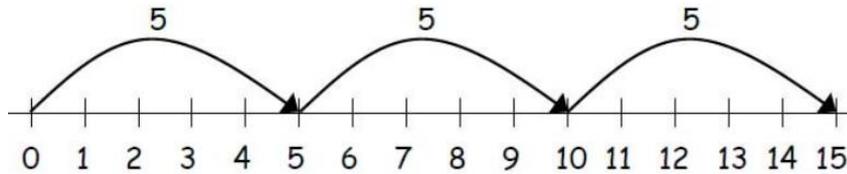
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Step 11: working with number lines

By showing pupils a range of different representations, you can help them sharpen their thinking, make connections and deepen conceptual understanding.

The final representation we are going to introduce is the number line.

Display a number line on the whiteboard. [You can use the Step 11 power point.](#)



Discuss what the pupils notice about the representation and what it might mean. You will need to introduce some new language to describe what the number line represents. We talk about jumps - the size of the jumps, the number of jumps and the total distance jumped.

[Find the Step 11 worksheet.](#)

Ask pupils to pair each number line with the matching FunKey Card. For more challenge, ask them to draw a bar model and/or a part-whole model which matches the number line, or a number line which shows the commutative fact.

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Step 12: play Five Dice

To conclude this sequence of activities, play this fun game which practises the language of multiplication (and revises place value and column addition skills!)

[Download the score sheet here.](#)

Aim of the game: to get the most points at the end of six turns each.

Ideally play in pairs or in a group of 3. If you have a larger group, pupils spend too much time watching and not enough time doing maths.

How to play

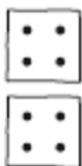
Each player gets six turns. Each turn is a maximum of three rolls.

Each turn, the player collects a chosen dice score. Players can collect dice scores in any order.

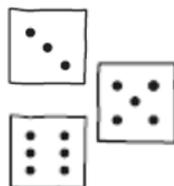
Example

Player 1 rolls all five dice. They look at the scores and decide to collect fours. They keep the two dice showing four and roll the other three dice again.

KEEP

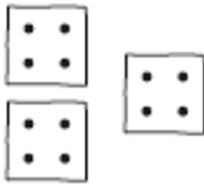


Roll Again

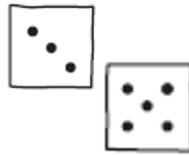


On their second roll, Player 1 rolls another four. They keep this four and roll the other two dice one more time.

KEEP



Roll Again



This time they get no more fours. They have had three rolls, so their turn is over, and they record their score as $3 \times 4 = 12$.

FIVE DICE

- Six turns
- Three rolls per turn
- Highest score wins

Roll	x fact	Score	
		10	1
	3×4	1	2
TOTAL			

Note that on the score card they show what they rolled by marking the dice images. This means that their work can be reviewed for understanding.

When Player 1's first turn is over, Player 2 rolls.

FIVE DICE

- Six turns
- Three rolls per turn
- Highest score wins

Roll	x fact	Score	
		10	1
	2×1		2
	4×2		8
	2×3		6
	3×4	1	2
	4×5		20
	5×6		30
TOTAL		7	8

When each player has had six turns and has a scored for each dice score, players add up their scores. Highest score wins.

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Congratulations!
You have now completed the activities!

The next unit to work through is [Recall 1](#).

If you have questions, or want advice, email us on hello@funkeymaths.com or contact us on Twitter [@FunKeyMaths](https://twitter.com/FunKeyMaths) where we also post details of new games, activities and resources.